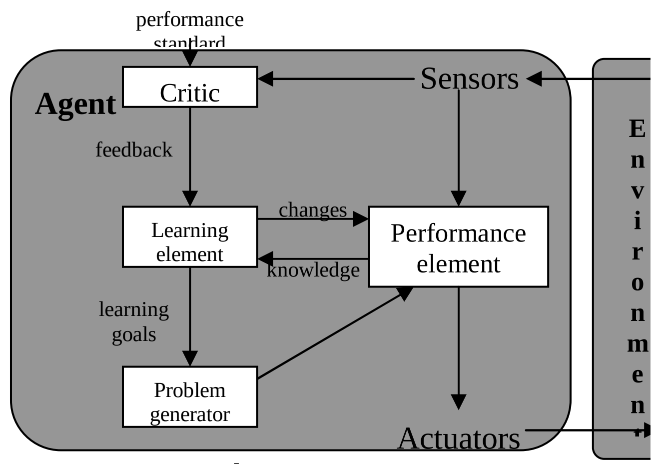
Session 3

I. Questions (Homework/LISP/Class)

AIMA

- What is the AIMA library.
- How can you use it

II. Issues Not In Class



learning agent

Learning – the process of modification of each component of an agent to make the components agree closer with the available feedback thereby improving the agent's performance.

- **learning element** – responsible for making improvements
- **performance element** – responsible for selecting external actions… the agent being modified.
- **critic** – provides feedback on the agent's performance and suggests improvements.
    - **performance standard** – a *fixed* measure of agent's performance.
        - distinguishes the *reward* in the percept by providing direct feedback on quality of agent's performance.
- **problem generator** – suggests actions that will lead to exploration.

Bidirectional Search – simultaneous searches from the initial state forward and from the goal state backwards that stop when the 2 searches meet. Encouraged by the fact that $b^{d/2} + b^{d/2} \ll b^d$

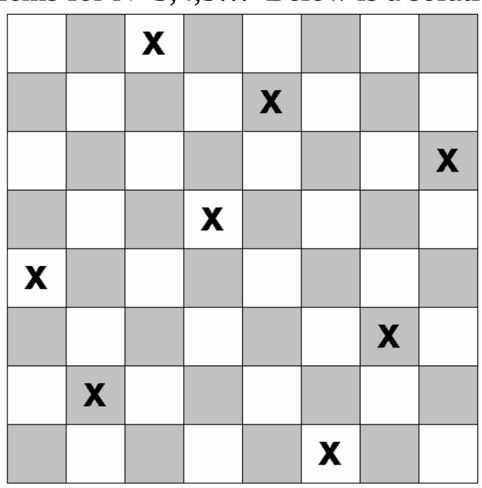
- *complete* & *optimal* (with uniform step costs) if both algorithms are BFS.
- Checking a node for membership in the other search tree can be done in constant time via a hash table, but requires that 1 search tree be in memory.
    - Time-complexity: $O\left(b^{d/2}\right)$ Space-complexity: $O\left(b^{d/2}\right)$
- Bidirectional search requires that the **predecessors** of a node be efficiently computable:
    - Easy when actions are *reversible*. Otherwise…
- To deal with several (explicitly listed) goal states, we make them all have a successor of a single *dummy goal state.*

III. N-QUEENS

Russell's Code

What is N-queens?

- Suppose we have N queens on a chess board of $N \times N$ squares.
    - Queens are allowed to move in any straight vertical, horizontal, or diagonal line indefinitely across the board to capture another piece.
    - We want to place N of them on the board so that no queens can be captured in a single move.
- What is the problem description (PEAS)? Okay, maybe this problem is a bit simple for the rigors of PEAS, but it's a good habit to always write out your problem description first.
    - Environment → Hmmm… the chess board and the queens.
    - Actions → Placing the queens.
    - Sensors → Rule checks that ensure no queens are in danger.
    - Performance Measure → Number of queens that can be captured in one move.
- How do we solve the problem?
    - Work problems for N=3,4,5… Below is a solution for N = 8[*]



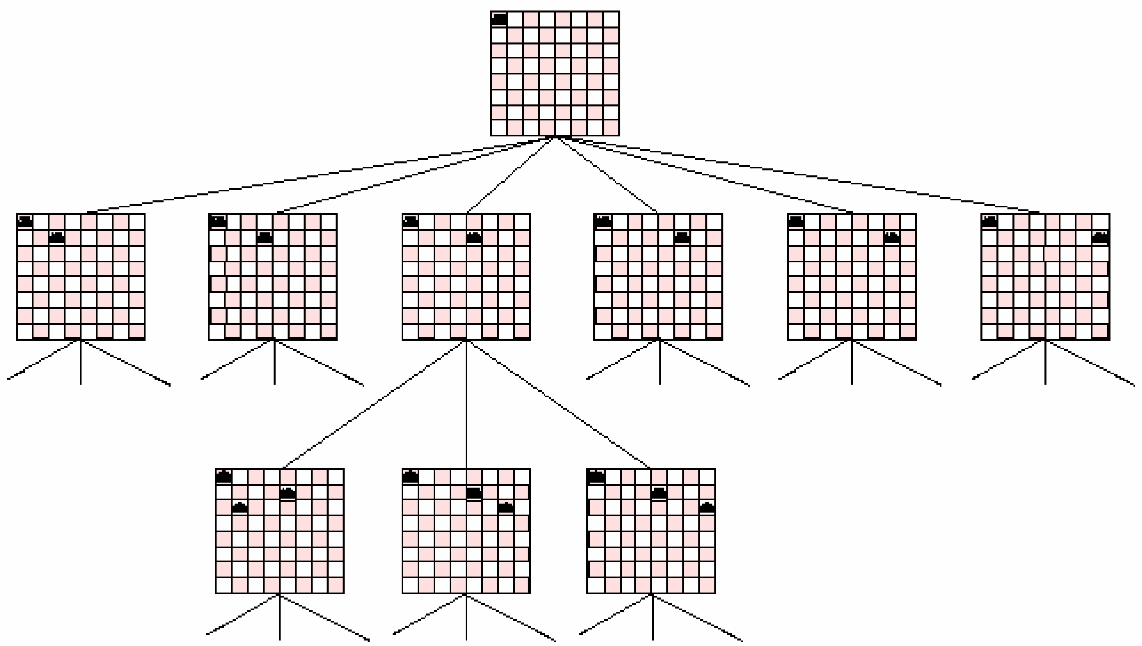
[*] The following image was taken from http://www.eudoxus.com/mp9609f1.gif

All Unique Solutions to the 8-Queens Problem†

Sol.Nbr.	Row 1	Row 2	Row 3	Row 4	Row 5	Row 6	Row 7	Row 8
1	1	5	8	6	3	7	2	4
2	1	6	8	3	7	4	2	5
3	2	4	6	8	3	1	7	5
4	2	5	7	1	3	8	6	4
5	2	5	7	4	1	8	6	3
6	2	6	1	7	4	8	3	5
7	2	6	8	3	1	4	7	5
8	2	7	3	6	8	5	1	4
9	2	7	5	8	1	4	6	3
10	3	5	2	8	1	7	4	6
11	3	5	8	4	1	7	2	6
12	3	6	2	5	8	1	7	4

† Table taken from http://www.durangobill.com/N_Queens.html

Uninformed Strategies[‡]



- Which uninformed strategy would be ideal for N-Queens?
  - **Breadth-First Search** – A bad idea for this problem. We are guaranteed to expand all nodes of depth less than *N* nodes. We'll never reach any goals until *N*-th level.
  - **Uniform Cost Search** – Not worth mentioning… no costs on our edges.
  - **Depth-Limited Search** – Ideal for this problem. ALL GOALS are at depth *N* so we can halt search there! Moreover Goals are *Dense*.
  - **Bidirectional – Problem** → formulating a goal state is hard in this case. If you knew the goal state, you've already solved the problem!
    - The states are cumulative (encapsulating all previous states) since we need to know the entire path to check whether we're in a goal.
    - However, we could have a global state and specify from both directions. BUT this is equivalent to any other ordering of piece placement – The placements are COMMUTATIVE.

In-depth look at problem

- It might be ridiculous to place the columns (rows) in order from left to right (top to bottom). What if other orders of placement were more efficient?
  - Not so bad actually. If we always were going in a left to right placement order, we should continue to do so.
  - The columns with the most constraints on their values are the leftmost since eventually diagonals run off the board.

[‡] Image taken from http://maven.smith.edu/~thiebaut/transputer/chapter9/chap9-4.html

- What are the simplest facts we can glean from the game?
  - Every queen must have it's own column… but every queen must have it's own row as well. If we think about this for a second, this means that every feasible N-queens solution must be a permutation of the list: $\langle 1, 2, \ldots, N \rangle$.
    - A **<u>permutation</u>** of a list is another list with the same elements in a different order!

      e.g. N=4 $\pi_a = \langle 2, 4, 1, 3 \rangle$
  - Now to incorporate a <u>diagonal constraint</u>. This can be formulated mathematically as $\forall s < t \quad |\pi(s) - \pi(t)| \neq t - s$.
  - Thus we have a way to write this problem mathematically; it must be a permutation that obeys the above constraint.

A* Search

- First a little book keeping about yesterdays lecture.
- **Consistent (Monotonic) Heuristic** – h(n) is not more than the cost through n to n' plus h(n'). Thus, a general triangle inequality:

$$h(n) \leq c(n, a, n') + h(n')$$

- Can A* do the job efficiently?
  - Heuristic Functions (In the Incremental Formulation).
    - Choosing a column to place. As discussed above, choosing a column to place is simple. Leftmost is probably *most constrained*.
    - Choosing a value for the column. Probably want a value that *limits the fewest other columns*.
      - This fails! All values have the same limitations. If we move on diagonal off the board we bring another one on.

Incremental vs Complete Formulation

- Which formulation is most convenient for the N-queens problem – incremental or complete.
  - <u>incremental formulation</u> – variables are assigned one at a time such that the assignment remains consistent.
    - Allows us to simply start with an empty board and add queens one column at a time – similar to the human approach.
    - Leads to a lot of backtracking (we come to final columns and realize there are no legal placements).
  - <u>complete-state formulation</u> – all variables are assigned initially and changed incrementally in attempts to make the assignment consistent.
    - valid since the path by which a solution is reached is irrelevant.
    - NO BACKTRACKING
      - When we have illegal queens, we simply move a single queen to remove possible captures.
  - ***Why do some problems fit well into incremental formulations and others into complete formulations?***

A*-Incremental N-Queens

- What are the possible moves in this formulation?
  - Move a queen in its column?
  - Swap a pair of queens across columns!!!
- Now what are good heuristic function?
  - Number of Queens in Conflict → overestimates.

Local Search Solutions to N-Queens

- Simulated Annealing – why?
- Genetic Algorithm

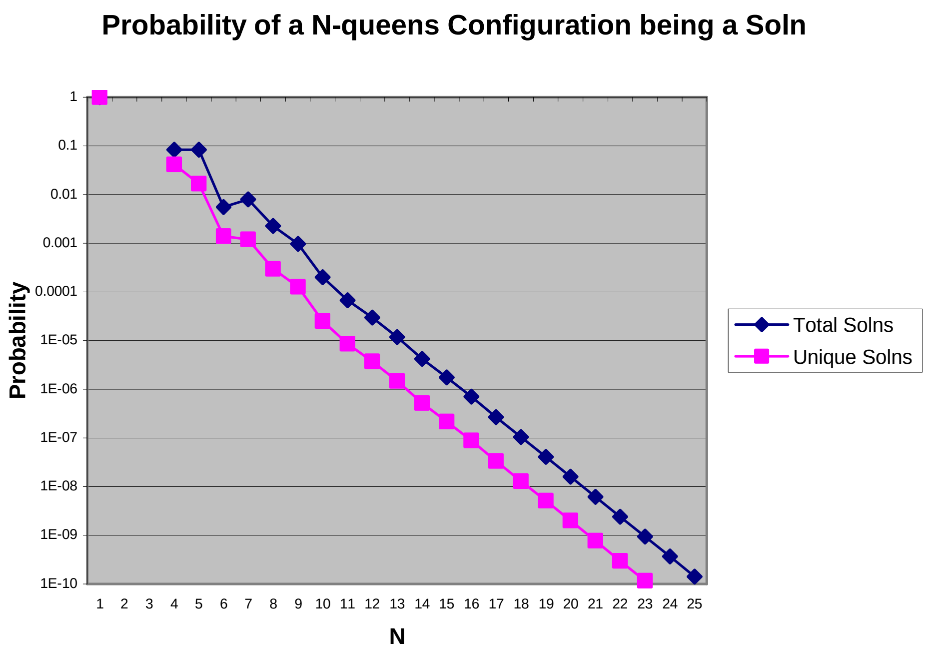
CSP Solutions

TBD next time

Solution Density

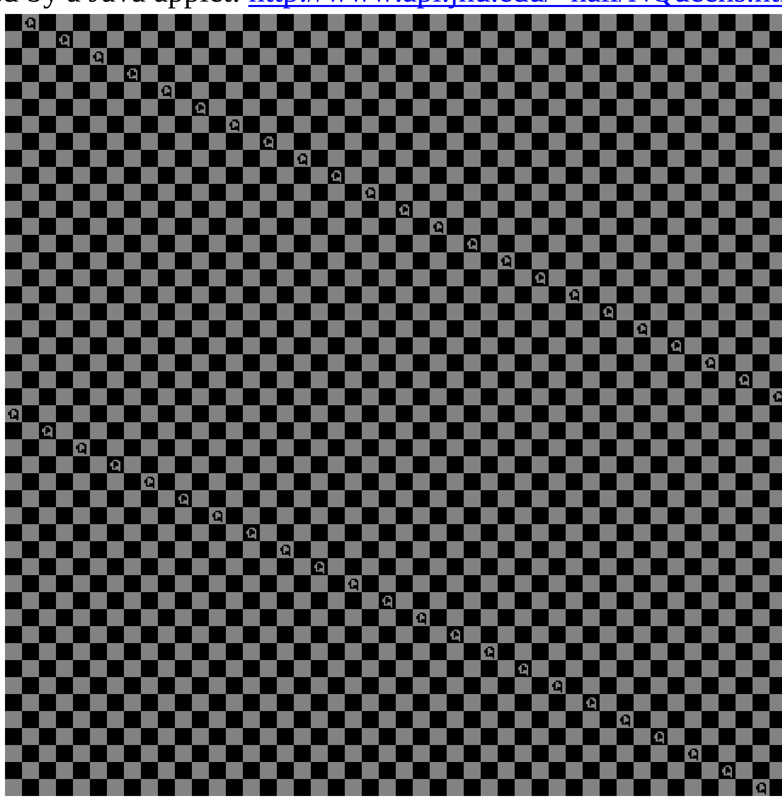
How Common are the N-queens solutions? The following table came from http://www.durangobill.com/N_Queens.html and shows the number of solutions (and unique solutions) along with their probabilities. *These probabilities are "inflated" in that I assumed the queens each had to be in separate rows or columns (N! such configurations) whereas, there are far more dumb solutions (N^2 choose N ~ $O(N^{2N})$).*

Order ("N")	Ordinary Queens Total Solutions	Ordinary Queens Unique Solutions	Probability of Total Solutions	Probability of Unique Solutions
1	1	1	1	1
2	0	0	0	0
3	0	0	0	0
4	2	1	0.083333333	0.041666667
5	10	2	0.083333333	0.016666667
6	4	1	0.005555556	0.001388889
7	40	6	0.007936508	0.001190476
8	92	12	0.002281746	0.000297619
9	352	46	0.000970018	0.000126764
10	724	92	0.000199515	2.53527E-05
11	2,680	341	6.71397E-05	8.54277E-06
12	14,200	1,787	2.9645E-05	3.73068E-06
13	73,712	9,233	1.18374E-05	1.48273E-06
14	365,596	45,752	4.19366E-06	5.2481E-07
15	2,279,184	285,053	1.74293E-06	2.17985E-07
16	14,772,512	1,846,955	7.06049E-07	8.82748E-08
17	95,815,104	11,977,939	2.6938E-07	3.36755E-08
18	666,090,624	83,263,591	1.04038E-07	1.30051E-08
19	4,968,057,848	621,012,754	4.08406E-08	5.10512E-09
20	39,029,188,884	4,878,666,808	1.60422E-08	2.00529E-09
21	314,666,222,712	39,333,324,973	6.15894E-09	7.69869E-10
22	2,691,008,701,644	336,376,244,042	2.39413E-09	2.99267E-10
23	24,233,937,684,440	3,029,242,658,210	9.3741E-10	1.17176E-10
24	227,514,171,973,736	?	3.66693E-10	
25	2,207,893,435,808,350	?	1.42342E-10	



How easy are solutions?

It turns out, for big N, solutions to the N-queens no longer look like intricate puzzles with clever tricks – they look like the simplest lines we could think of (Below is a solution for N=46 produced by a Java applet: http://www.apl.jhu.edu/~hall/NQueens.html).



Russell's Code for N-Queens

```
;;; -*- Mode: Lisp; Syntax: Common-Lisp; -*-

;;; n-queens as a search problem.
;;; We give both an incremental formulation [2e p 66]
;;; and a complete-state formulation [2e p 110-111].
;;; We also provide the methods required for applying
;;; genetic algorithms to the complete-state formulation.

;;;; Incremental formulation: add one queen at a time, avoiding illegal choices.

(defstruct (nqueens-incremental-problem
            (:include problem) (:constructor create-nqueens-incremental-problem))
  n  ;;; the number of queens (n x n board)
  )

(defun make-nqueens-incremental-problem (&key (n 8))
  "Returns an nqueens problem instance with an empty board. In general,
   a state is an n-element vector of queen positions, one per column."
  (create-nqueens-incremental-problem
   :n n :initial-state (make-sequence 'vector n :initial-element nil)))

(defmethod copy-state ((state vector)) (copy-seq state))

(defmethod goal-test ((problem nqueens-incremental-problem) state)
  "Return true if all queens have been placed, i.e., last queen is non-nil."
  (elt state (1- (nqueens-incremental-problem-n problem))))

(defmethod actions ((problem nqueens-incremental-problem) state)
  "Generate the possible moves from an nqueens-incremental state.
   A move is simply the row position of the queen in the next column."
  (let ((n (length state))
        (next-col (position nil state))
        (actions nil))
    ;;; For each possible square, check if attacked by previously placed queens
    (loop for row from 0 to (1- n) do
      (unless (some #'(lambda (col)
                        (let ((q (elt state col)))
                          (or (= q row) (= (- next-col col) (abs (- q row))))))
                    (iota next-col))
        (push row actions)))
    actions))

(defmethod result ((problem nqueens-incremental-problem) action state)
  (let ((outcome (copy-state state)))
    (setf (elt outcome (position nil state)) action)
    outcome))

(defmethod h-cost ((problem nqueens-incremental-problem) state)
  "Number of unfilled columns."
  (let ((next-col (position nil state)))
    (if next-col (- (nqueens-incremental-problem-n problem) next-col) 0)))

(defun print-nqueens-state (state)
  "Print out nqueens board state."
  (let ((n (length state)))
    (loop for j from (1- n) downto 0 do
      (format t "~%")
      (loop for i from 0 to (1- n) do (format t (if (= (elt state i) j) "Q " ". "))))))


;;;; Complete-state formulation: start with all queens on
;;;; the board, pick any queen and move it in its column.

(defstruct (nqueens-complete-problem
            (:include problem) (:constructor create-nqueens-complete-problem))
  n        ;;; the number of queens (n x n board)
  )
```

```lisp
(defun make-nqueens-complete-problem (&key (n 8))
  "Returns an nqueens problem instance with all n queens placed
   randomly, one per column."
  (create-nqueens-complete-problem
   :n n :initial-state (random-nqueens-complete-state n)))

(defmethod goal-test ((problem nqueens-complete-problem) state)
  (zerop (h-cost problem state)))

(defmethod actions ((problem nqueens-complete-problem) state)
  "Generate the possible moves from a complete nqueens state.
   A move is simply the column and the new row for that queen."
  (let ((n (length state))
        (actions nil))
    ;;; For each column, generate all other rows but the current one
    (loop for col from 0 to (1- n) do
      (let ((q (elt state col)))
        (loop for row from 0 to (1- n) do
          (unless (= row q) (push (list col row) actions)))))
    actions))

(defmethod result ((problem nqueens-complete-problem) action state)
  "Return a new state with the specified queen moved to the specified square."
  (let ((outcome (copy-state state)))
    (setf (elt outcome (first action)) (second action))
    outcome))

(defmethod h-cost ((problem nqueens-complete-problem) state)
  "Number of pairs of queens attacking each other."
  (let ((n (length state))
        (sum 0))
    (loop for i from 0 to (- n 2) do
      (loop for j from (1+ i) to (- n 1) do
        (let ((delta (- (aref state i) (aref state j))))
          (when (or (= delta 0) (= (abs delta) (- j i)))
            (incf sum)))))
    sum))

(defun random-nqueens-complete-state (n)
  "Return a random complete state with n queens, one per column."
  (let ((state (make-sequence 'vector n)))
    (loop for i from 0 to (1- n) do
       (setf (elt state i) (random n)))
    state))



;;;; Methods for genetic algorithms applied to complete-state nqueens

(defmethod GA-encode ((problem nqueens-complete-problem) state)
  "Encode state as a sequence - already in that form."
  state)

(defmethod GA-decode ((problem nqueens-complete-problem) individual)
  "Decode state from sequence - already in that form."
  individual)

(defmethod GA-alphabet ((problem nqueens-complete-problem))
  "Return the list of characters used in sequence form - 0 through n-1."
  (iota (nqueens-complete-problem-n problem)))

(defmethod fitness ((problem nqueens-complete-problem) individual)
  "Return the number of non-attacks between queens."
  (let ((n (length individual)))
    (- (/ (* n (- n 1)) 2) (h-cost problem individual))))
```

Appendix: Problem

```
;;; -*- Mode: Lisp; Syntax: Common-Lisp; -*- File: problems.lisp

;;;; Defining Problems

(defstruct problem
  "A problem is defined by the initial state, successor function,
   goal test, and path cost (defined, in turn, by step cost). [2e p 62]"
  (initial-state (required)) ; A state in the domain
  )

;;; When we define a new subtype of problem, we need to specify eeither
;;; 1) a SUCCESSOR-FN method; or
;;; 2) ACTIONS and RESULT methods.
;;; If one or the other is not done, an infinite loop will result!
;;; We may also need to define methods for GOAL-TEST, H-COST, and
;;; STEP-COST, but they have default methods which may be appropriate.
;;; In addition, there is a technicality: states and actions require
;;; hash keys, although a default is provided that often works (see below).

(defmethod successor-fn ((problem problem) state)
  "Return a list of (action . state) pairs that can be reached from this state."
  (mapcar #'(lambda (action) (cons action (result problem action state)))
                  (actions problem state)))

(defmethod actions ((problem problem) state)
  "Return an list of actions possible in this state;
   use this default method only if successor-fn is independently defined!"
  (mapcar #'car (successor-fn problem state)))

(defmethod result ((problem problem) action state)
  "Return the state resulting from executing action in state;
   use this default method only if successor-fn is independently defined!"
  (cdr (assoc action (successor-fn problem state)
                      :test #'(lambda (a1 a2)
                                              (equalp (action-hash-key problem a1)
                                                         (action-hash-key problem a2))))))

(defmethod sequence-result ((problem problem) action-sequence state)
  "Return the state resulting from executing action-sequence in state.
   Useful for checking that a proposed solution sequence achieves the goal."
  (if (null action-sequence)
      state
    (sequence-result problem (rest action-sequence)
                               (result problem (first action-sequence) state))))

(defmethod successor-states ((problem problem) state)
  "Return a list of states that can be reached from this state.
   This ignores actions, and is appropriate only for offline local search."
  (mapcar #'(lambda (action) (result problem action state))
                  (actions problem state)))



(defmethod goal-test ((problem problem) state)
  "Return true or false: is this state a goal state?"
  (declare-ignore state)
  (required))

(defmethod step-cost ((problem problem) state1 action state2)
  "The cost of going from state1 to state2 by taking action.
  This default method counts 1 for every action.  Provide a method for this if
  your subtype of problem has a different idea of the cost of a step."
  (declare-ignore state1 action state2)
  1)
```

```lisp
(defun path-cost (problem action-sequence &optional (state (problem-initial-state problem)) (cost 0))
  "Return the sum of step costs along the given action sequence."
  (if (null action-sequence)
      cost
    (let ((next-state (result problem (first action-sequence) state)))
      (path-cost problem (rest action-sequence) next-state
                             (+ cost (step-cost problem state (first action-sequence) next-state))))))

(defmethod h-cost ((problem problem) state)
  "The estimated cost from state to a goal for this problem.
  If you don't overestimate, then A* will always find optimal solutions.
  The default estimate is always 0, which certainly doesn't overestimate."
  (declare (ignore state))
  0)

;;; The ability to generate a single random successor,
;;; rather than all successors at once, is important for
;;; local search algorithms in domains with large state
;;; representations and/or many successors.

(defmethod random-successor ((problem problem) state)
  "Return (a . s) for a random legal action a and outcome s."
  (let ((action (random-action problem state)))
    (cons action (result problem action state))))

(defmethod random-successor-state ((problem problem) state)
  "Return the outcome s of a random legal action."
  (result problem (random-action problem state) state))

(defmethod random-action ((problem problem) state)
  "Return a random legal action in state; typically this
   method must be defined specially for each domain."
  (random-element (actions problem state)))



;;; Hash keys for states and actions.
;;; States are hashed in the graph search algorithms; both states and actions
;;; are hashed in the enumerated-problem class. Two states or actions represented
;;; by complex data structures may not be EQUALP if the representation
;;; is not canonical, so we must define hash keys for them.
;;; For example, moves in backgammon can be written in any permutation
;;; and still be the "same" move. However, this situation is rare.
;;; In most cases, the state or action representation serves as its own hash key.

(defmethod state-hash-key ((problem problem) state)
  "Key to be used to hash the state; identical states must have EQUAL keys.
   Default is the state itself, i.e., assume a canonical representation."
  state)

(defmethod action-hash-key ((problem problem) action)
  "Key to be used to hash actions; identical actions must have EQUAL keys."
  action)
```