adversarial search (games) – competitive multiagent environments (agent’s have conflicting goals). In particular, adversarial search is mixture of search and game theory. The typical game is a deterministic, turn-taking, two-player zero-sum game of perfect information. These games are a sequence of decisions that reach a terminal state.

Below is a partial game tree for tic-tac-toe:

- **game tree** – a representation that represents all legal sequences of decisions.
  - root – the initial state of the game (with a starting player).
  - (internal) nodes – represents decision made by one of the players. The node is labeled by the player making the decision (Max/Min).
  - edges – legal choices for a given decision in the tree. These are specified by a successor function that lists legal (move, state) pairs.
  - terminal node – an ending of the game giving a utility to each player.

- **utility function** – maps a terminal state to a value.

- **optimal strategy** – a contingent strategy that leads to an outcome at least as good as any other strategy by assuming the opponent is infallible.
  - minimax algorithm – finds an optimal strategy by depth-first exhaustive search which annotates each node of the tree with a minimax-value:

\[
\text{minimax-value}(n) = \begin{cases} 
\text{utility}(n) & n \in \text{Terminal} \\
\max_{s \in \text{child}(n)} \text{minimax-value}(s) & n \in \text{MAX} \\
\min_{s \in \text{child}(n)} \text{minimax-value}(s) & n \in \text{MIN}
\end{cases}
\]

- **alpha-beta pruning** – a modified minimax search that prunes branches that cannot influence the final result.
  - \( \alpha \) – the maximum value so far at any choice point along the path for MAX
  - \( \beta \) – the minimum value so far at any choice point along the path for MIN
Stopping search prematurely – time limits prevent full exploration of the game tree.

- **evaluation function** – a heuristic for accessing the utility of a nonterminal game state; that is, it returns an estimate of the expected value of a state.
  - **features** – elements of the state that indicate its strength.
    - features form categories (equivalence classes) among states.
    - many evaluation functions combine numerical contributions from each feature as an estimate (e.g. weighted linear function).
- **cutting-off search** – determine a reasonable time to stop search (e.g. iterative deepening explores deeper until time elapses).
  - evaluation function should only be applied to positions that are unlikely to have major changes in the near future (quiescent).
  - **horizon effect** – an unavoidable damaging move looms on the horizon.

Games of Chance

- **chance nodes** – nodes (denoted by circles) indicating an element of chance is introduced and arcs from this node are probabilistic transitions
  - The minimax algorithm is identical & chance nodes are expected values:
    \[
    \text{expectiminimax} (n) = \sum_{s \in \text{child}(n)} P(s) \cdot \text{expectiminimax}(s) \quad n \in \text{Chance}
    \]
  - In games of chance, the evaluation function must be a positive linear transform of the probability of winning from a position.
  - Pruning of chance nodes is possible if bounds can be placed on possible values (thereby bounding the possible values of the average).

Games of Chance with imperfect information

- **averaging over clairvoyancy** – the strategy of computing optimal moves by averaging over possibilities for the unseen variables.
  - This strategy is flawed as it assumes all future uncertainty will have disappeared by the time the future is reached.
  - Thus, the strategy never makes moves that seek to reveal information.
- **belief states** – games states are replaced by possible states along with their corresponding probabilities.
- *In games of imperfect information, it's best to reveal as little as possible, often by acting unpredictably.*