

Propositional Puzzles

In Class Example

Use Propositional Logic to show that, in the game, “Heads I win, Tails you lose”, I always win¹

1. Make our objects:
H – heads T – tails I – I win Y – you win
2. State your rules:
 - a. $H \Rightarrow I$ and $T \Rightarrow \neg Y$

Is that all? Don't forget, you must specify implicit rules, too! The system doesn't know that heads and tails are mutually exclusive.

- b. $H \otimes T$ and $I \otimes Y$
3. Convert to CNF
 $\neg H \vee I$ $\neg T \vee \neg Y$ $(H \vee T) \wedge (\neg H \vee \neg T)$
 $(I \vee Y) \wedge (\neg I \vee \neg Y)$
4. We want to prove I , so insert the literal $\neg I$ for the proof by contradiction. Now start resolving clauses:
 - a. $\neg T \vee \neg Y$ and $H \vee T \rightarrow H \vee \neg Y$
 - b. $\neg H \vee I$ and $H \vee \neg Y \rightarrow I \vee \neg Y$
 - c. $\neg I$ and $I \vee \neg Y \rightarrow \neg Y$
 - d. $I \vee Y$ and $\neg Y \rightarrow I$
 - e. I and $\neg I \rightarrow \{\}$ -- we have a *contradiction* $\rightarrow I$ is true.

Solve the Mystery

The following example was taken from the following website:

<http://logic.stanford.edu/classes/cs157/2005fall/notes/chap05.pdf>

- There are three suspects for a murder: Adams, Brown, and Clark.
- Adams says “I didn't do it. The victim was old acquaintance of Brown's. But Clark hated him.”
- Brown states “I didn't do it. I didn't know the guy. Besides I was out of town all the week.”
- Clark says “I didn't do it. I saw both Adams and Brown downtown with the victim that day; one of them must have done it.”
- Assume that the two innocent men are telling the truth, but that the guilty man might not be.
- Write out the facts as sentences in Propositional Logic, and use propositional resolution to solve the crime.

¹ Example taken from <http://logic.stanford.edu/classes/cs157/2005fall/notes/chap05.pdf>